Math 402 Chapter 16 Take Home Exam

For problems #1 – 3 the curve c is the path traced out by \( r(t) = t^2 \mathbf{i} + t^3 \mathbf{j} \) on \( 0 \leq t \leq 1 \).

1. Evaluate the line integral \( \int_c x^2 dy + 2y dx \).

2. Evaluate the line integral \( \int_c x ds \).

3. Evaluate the line integral \( \int_c \mathbf{F} \cdot d\mathbf{r} \) where \( \mathbf{F} = (2x + 3y) \mathbf{i} + (5x - 4y) \mathbf{j} \)

For problems # 4 - 6 \( f(x, y) = 2y(x^2 + y^2)^{3/2} \) and its gradient field \( \mathbf{F} = \nabla f = 2 \sqrt{x^2 + y^2} \left( 3xy, x^2 + 4y^2 \right) \).

4. Compute \( \nabla \cdot \mathbf{F} \) and \( \nabla \times \mathbf{F} \).

5. Evaluate \( \int_c \mathbf{F} \cdot d\mathbf{r} \) directly, where \( c \) is the portion of the circle of radius 2 and centered at the origin that is in the second quadrant and is oriented counter-clockwise with respect to the origin.

6. Evaluate \( \int_c \mathbf{F} \cdot d\mathbf{r} \) by applying the fundamental theorem of line integrals.

7. Find a scalar function \( f(x, y) \) so that \( \mathbf{F} = \nabla f \) where \( \mathbf{F} = (xy \cosh xy + \sinh xy) \mathbf{i} + (x^2 \cosh xy) \mathbf{j} \) (Such a function exists since \( \mathbf{F} \) is defined and differentiable throughout \( \mathbb{R}^2 \) and \( \nabla \times \mathbf{F} = \mathbf{0} \) )

8. A surface of revolution is obtained by revolving the curve \( z = \sin x \) (in the x-z plane) about the z-axis. The resulting surface can be parameterized by \( \mathbf{r}(r, \theta) = r \cos \theta \mathbf{i} + r \sin \theta \mathbf{j} + \sin r \mathbf{k} \). Find \( \mathbf{r}_r \) and \( \mathbf{r}_\theta \) then use the results to determine \( d\mathbf{S} \) and \( dS \).

9. Verify Green's Theorem for \( \mathbf{F} = (7x + 3y) \mathbf{i} + (11x - 6y) \mathbf{j} \) and where \( c \) is the circle \( (x - 2)^2 + y^2 = 1 \) oriented counter-clockwise.

10. Use the Divergence Theorem to evaluate \( \iint_S \mathbf{F} \cdot d\mathbf{S} \) where \( \mathbf{F} = (4x + z^2 y) \mathbf{i} + (x^3 z - 4y) \mathbf{j} + (7x^3 y + 3z^2) \mathbf{k} \) and \( S \) is the surface of the solid cylinder described by \( 0 \leq r \leq 1 \) and \( 0 \leq z \leq 2 \).