

Objectives--understand the concepts and calculate:

- Future Value (or compound amount) of a single amount
- Present Value of a single amount
- Future Value of an annuity
- Present Value of an annuity
- Present Value of notes and bonds

Basic Concepts and Assumptions:

- Money invested today will grow to a larger amount at a later date (the future). The growth in the money is called interest and it is periodically added to the original investment (principal) so that it is treated as principal for future interest earning calculations. Because of the adding of each period's interest to the principal, each subsequent interest amount is larger than the previous. This is the concept of compounding interest. Basically, the future value of an amount (principal) is equal to the original principal plus the interest earned.
- As a result of the previous concept money in the future is equal to a smaller amount of money today (the present). It is essentially the reverse of the above. To find out how much money you would need today (present value) so that it accumulates to a given amount in the future (future value) one would deduct the interest. But remember the interest has to be deducted using the same concept of compounding. This deducting or taking away of interest is called "discounting". Basically, the present value of a future amount is equal to that future amount less the interest discounted.
- Because simple interest calculations assume that the interest earned is not added periodically but is added all at once at the end, the future value of an amount using simple interest would result in a smaller future value. Similarly, the present value of a future amount at simple interest would be larger than at compound interest because less interest would be deducted.

Method of Solving Problems:

Time Value Money (TVM) formulas, tables based upon the formulas, business calculators with built in functions or special software programs like TValue and Excel can be used in calculating compound interest (TVM) problems. This paper will focus on how to make the calculations using Tables because it is the only method that is available to all students in the class.

Tables:

The rates shown across the top of the tables are rates per period--nominal or annual rates divided by the number of compounding periods per year. The periods shown down the left side of the tables represent the total number of compounding periods—the number of periods per year times the number of years.

For example, the Future Value of 1 table is a set of factors that represent the future value of one dollar (\$1.00) at various interest rates for a number of compounding periods. Notice that all the values are greater than one. This is because the future value will be larger since it is the present value plus

compound interest. Compare this to all factors in the Present Value of 1 table. It's a set of factors that represent the present value of one dollar (\$1.00) at various interest rates for a number of compounding periods. Notice that all the factors are less than one. This is because the present value will be smaller than the compounded future value since it is the future amount less the interest.

Timelines:

Using timelines is a good way to illustrate (visualize) what amounts are given (known) and what is to be determined (unknown). It is critical that you know what kind of problem you are trying to solve because there are four different tables to choose from. Be sure to look for key words and phrases in problems, such as "now," "today," "in the present," and "in the future," "in x years", "how much later."

Avoiding Getting Confused About Which Table To Use:

When solving for a future value of an amount use the Future Value of 1 table. The table factors are all greater than one (1.0000) because the answer is going to be equal to the Principal plus the interest.

When solving for a present value of an amount use the Present Value of 1 table. The table factors are all less than one (1.0000) because the answer is going to be equal to the Compounded Amount less the interest.

Future Value of an Amount (aka Future Value of 1, Future Value of a Dollar or Compound Amount)--the total amount of principal and accumulated interest at the end of a loan or investment.

- Step 1. Determine the number of compounding periods. (This is the total number of years multiplied times the number of time periods per year.)
- Step 2. Determine the interest rate per period. (This is the nominal or annual rate divided by the number of time periods per year.)
- Step 3. Find the future value of an amount table factor. (This is the intersection of the number of compounding periods and interest rate per period.)
- Step 4. Calculate the Future value. (The answer is determined by multiplying the table factor from Step 3 times the Principal amount.)

Present Value of an Amount (aka Present Value of 1, Present Value of a Dollar or Present Value of a Single Amount)--is the amount of money that must be deposited today, at compound interest, to provide a specified lump sum of money in the future.

- Step 1. Determine the number of compounding periods. (Same as previous example.)
- Step 2. Determine the interest rate per period. (Same as previous example.)
- Step 3. Find the present value of an amount table factor. (Same as previous example.)
- Step 4. Calculate the Present Value. (The table factor multiplied times the future amount or principal.)

Annuities:

An annuity is a series of equal payments (also known as rents) with one payment each period. The payments can be going out, such as a mortgage payment, or coming in, such as monthly retirement income. Tables can also be used in solving annuity problems. The Future Value of an Annuity of 1 is a set of factors that represent the future value of an annuity of one dollar (\$1.00) at various interest rates for

a number of compounding periods. In effect the sum of all the rents plus interest. Whereas the Present Value of an Annuity of 1 is a set of factors that represent the present value of an annuity of one dollar (\$1.00) at various interest rates for a number of compounding periods. In effect the sum of all the rents less the interest. Also note that all the factors in both tables are greater than 1 but the future value table factors are much greater than present value table factors. Again this is because the future value is plus interest whereas the present value is less the interest.

Future Value of an Annuity-- the lump sum which results from an annuity.

- Step 1: Calculate the interest rate per period for the annuity.
- Step 2: Determine the number of periods.
- Step 3: Find the Future Value of an annuity table factor.
- Step 4: Solve for the answer by multiplying the table factor times **one** rent. The annuity tables have already incorporated the number of rents in the factors given in the tables.

Present Value of an Annuity—the present value of an annuity is the “lump sum” that must be deposited today to provide a series of equal payments (an annuity) in the future.

- Step 1: Calculate the interest rate per period for the annuity.
- Step 2: Determine the number of periods of the annuity.
- Step 3: Find the Present Value of an Annuity factor.
- Step 4: Solve for the answer by multiplying the table factor times **one** rent.

Calculating the Present Value of Notes and Bonds that include periodic interest payments:

Independently solve for the present value of an amount and solve for the present value of an annuity, then add the answers together. In effect it is simply two separate present value calculations which are added together. Normally the interest rate to use in these types of calculations is the current “market” rate.